What you should learn from Recitation 9: Laplace Transforms

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April 6, 2014

Disclaimer

- The slides are intended to serve as records for a recitation for math 244 course. It should never serve as any replacement for formal lectures or as any reviewing material. The author is not responsible for consequences brought by inappropriate use.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

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Remark: The inverse Laplace transform exists and can be defined via complex functions, the theory of which could be seen in Calc5.

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Attention: Don't mess up with the signs! From the second term on, everything is negative.

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$$y'' - 3y' + 2y = e^t, y(0) = 0, y'(0) = 1$$

- Perform the Laplace transform: Let $Y(s) = \mathcal{L}(y(t))$, then

$$(s^2Y(s) - sy(0) - y'(0)) - 3(sY(s) - y(0)) + 2Y(s) = \frac{1}{s-1},$$

Putting in $y(0)$ and $y'(0)$:

$$(s^2 - 3s + 2)Y(s) - 1 = \frac{1}{s-1}$$

Solve for $Y(s)$:

$$Y(s) = \frac{s}{(s^2 - 3s + 2)(s-1)}.$$

Quiz Problem 2

- Break $Y(s)$ into partial fractions:

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So $B = -2$

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and the ODE is solved.

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$$f(t) = e^t \cos 3t.$$

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Therefore

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  $$\mathcal{L}(e^t e^{3it}) = \frac{s-1+3i}{(s-1)^2+9} = \frac{s-1}{(s-1)^2+9} + i\frac{3}{(s-1)^2+9},$$

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Homework Problem 6.2.17

Solve the IVP

$$y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1$$

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  The cover up method gives

$$A = (s^2 - 4s + 7)|_{s=1}$$

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Subtract the left-hand-side with $4/(s-1)^4$,

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$$A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4$$

Subtract the left-hand-side with $4/(s-1)^4$, one gets

$$\frac{s^2 - 4s + 7 - 4}{(s-1)^4}$$

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Cover-up method gives

$$B = (s-3)|_{s=1} = -2.$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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- Break $Y(s)$ into partial fractions (continued): So

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Again by subtraction one gets

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$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

Again by subtraction one gets

$$\frac{(s-3+2)}{(s-1)^3} = \frac{1}{(s-1)^2} = \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

Again by subtraction one gets

$$\frac{(s-3+2)}{(s-1)^3} = \frac{1}{(s-1)^2} = \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

Then immediately $C = 1$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So
$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$
Again by subtraction one gets
$$\frac{(s-3+2)}{(s-1)^3} = \frac{1}{(s-1)^2} = \frac{C}{(s-1)^2} + \frac{D}{s-1}$$
Then immediately $C = 1$ and $D = 0$.

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Then immediately $C = 1$ and $D = 0$.So

$$Y(s) = \frac{4}{(s-1)^4} - \frac{2}{(s-1)^3} + \frac{1}{(s-1)^2}.$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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$$Y(s) = \frac{4}{(s-1)^4} - \frac{2}{(s-1)^3} + \frac{1}{(s-1)^2}.$$

- Perform the inverse transformation.

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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$$Y(s) = \frac{4}{(s-1)^4} - \frac{2}{(s-1)^3} + \frac{1}{(s-1)^2}.$$

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula.

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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$$Y(s)$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) \quad = \quad \frac{4}{3!}\frac{3!}{(s-1)^4}$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) = \frac{4}{3!}\frac{3!}{(s-1)^4} - \frac{2!}{(s-1)^3}$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So
$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$
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$$Y(s) \quad = \quad \frac{4}{3!}\frac{3!}{(s-1)^4} - \frac{2!}{(s-1)^3} + \frac{1!}{(s-1)^2}$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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$$y(t)$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$\begin{aligned} Y(s) &= \frac{4}{3!}\frac{3!}{(s-1)^4} - \frac{2!}{(s-1)^3} + \frac{1!}{(s-1)^2} \\ y(t) &= \frac{4}{6}e^t t^3 \end{aligned}$$

Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s-3}{(s-1)^3} = \frac{-2}{(s-1)^3} + \frac{C}{(s-1)^2} + \frac{D}{s-1}$$

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Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula.

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula. By what you have learned in 6.3,

Homework Problem 6.2.25

Solve the IVP

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- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

$$y'' + y = t - tu_1(t).$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

$$y'' + y = t - tu_1(t).$$

- Perform the Laplace transform

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

$$y'' + y = t - tu_1(t).$$

- Perform the Laplace transform using the derivative formula and t-axis translation formula:

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

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- Perform the Laplace transform using the derivative formula and t-axis translation formula:

$$s^2 Y(s) + Y(s) = \frac{1}{s^2}$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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- Perform the Laplace transform using the derivative formula and t-axis translation formula:

$$s^2 Y(s) + Y(s) \quad = \quad \frac{1}{s^2} - e^{-s}\mathcal{L}(t+1)$$

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Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

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$$\begin{aligned} s^2 Y(s) + Y(s) &= \frac{1}{s^2} - e^{-s}\mathcal{L}(t+1) \\ &= \frac{1}{s^2} \end{aligned}$$

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$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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Homework Problem 6.2.25

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$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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Homework Problem 6.2.25

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$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued):

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform.

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2}$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1},$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

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$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1}, \frac{1}{s(s^2+1)}$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1}, \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1}, \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

Then

$$y(t)$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1}, \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

  Then

$$y(t) = (t - \sin t)$$

Homework Problem 6.2.25

Solve the IVP

$$y'' + y = \begin{cases} t & 0 \le t < 1 \\ 0 & 1 \le t < \infty \end{cases}, y(0) = 0, y'(0) = 0$$

- Perform the Laplace transform (continued): So after algebra,

$$Y(s) = \frac{1}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s^2} - \frac{e^{-s}}{(s^2+1)s}.$$

- Find the inverse Laplace transform. By whatever method you have,

$$\frac{1}{(s^2+1)s^2} = \frac{1}{s^2} - \frac{1}{s^2+1}, \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

Then

$$y(t) = (t - \sin t) + u_1(t)$$

Homework Problem 6.2.25

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Then

$$y(t) = (t - \sin t) + u_1(t)\left[t - 1 - \sin(t-1)\right.$$

Homework Problem 6.2.25

Solve the IVP

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Then

$$y(t) = (t - \sin t) + u_1(t)\left[t - 1 - \sin(t-1) + 1\right.$$

Homework Problem 6.2.25

Solve the IVP

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Then

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Homework Problem 6.2.25

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The End